INTERACTIONS IN INTERSECTING BRANE MODELS

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We calculate tree level three and four point scattering amplitudes in type II string models with matter fields localized at the intersections of D-brane wrapping cycles. The analysis of the three point amplitude is performed in the context of Yukawa couplings and it is seen that a natural mechanism for the generation of a mass hierarchy arises. The four point amplitude for fermions at the intersection of four independent stacks of D-branes is then determined.

1 Introduction

The intersecting brane scenario has been remarkably successful in producing semi-realistic models. For example, models similar to the Standard Model can be obtained [1,2,3,4,5,6,7,8] and viable constructions with N=1 supersymmetry have been developed [9,10,11,12,13]. Furthermore, they also provide a rather attractive topological explanation of family replication.

In this talk, we deal with the computation of the general three point and four point amplitudes of string states localised at the intersections of D6-branes wrapping $T^2 \times T^2 \times T^2$. Our calculations are based on and extend work presented in [14,15] and may easily be adapted to other scenarios involving intersecting branes. A more thorough discussion of the general four point amplitude and the generalisation to N-point amplitudes will be presented in [16], where a detailed list of references is also provided.

2 The general three point amplitude

We begin by discussing the three point amplitude in the context of Yukawa interactions.

2.1 Yukawa interactions

Due to our choice of internal space, the amplitude factorises into three identical contributions, one from each torus subfactor. Therefore, concentrating on a single torus, the string states are localised at the vertices of a triangle whose boundary consists of a single internal dimension from each of the D6-branes,

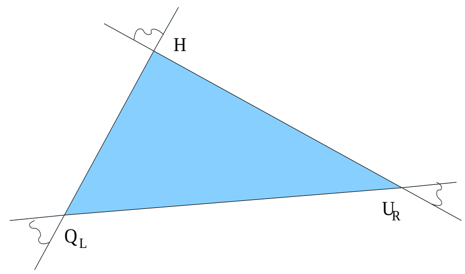

Figure 1. Yukawa interaction.

as depicted in figure 1. One would expect the amplitude to be dominated by an instanton, and thus be proportional to $e^{-\frac{1}{2\pi\alpha}A}$ where A is the area of the triangle. This will be borne out in the following calculation.

We denote the spacetime coordinates of a single torus by $X = X^1 + iX^2$ and $\bar{X} = X^1 - iX^2$. The bosonic field X can be split up into a classical piece, X_{cl}, and a quantum fluctuation, X_{qu}. The amplitude then factorises into classical and quantum contributions,

$$Z = \sum_{\langle X_{cl} \rangle} e^{-S_{cl}} Z_{qu}. \tag{1}$$

where,

$$S_{cl} = \frac{1}{4\pi\alpha}\left(\int d^2z(\partial X_{cl}\bar{\partial}\bar{X}_{cl} + \bar{\partial}X_{cl}\partial\bar{X}_{cl}\right). \tag{2}$$

X_{cl} must satisfy the string equation of motion and possess the correct asymptotic behaviour near the triangle vertices. This behaviour is determined using the analogy between open strings at brane intersections and closed strings on orbifolds, which we now discuss.

2.2 Boundary conditions and twist vertices

Consider an open string stretched between two intersecting D-branes at an angle $\pi\vartheta$. Solving the string equation of motion using the appropriate boundary conditions, we obtain the mode expansion,

$$\partial X(z) = \sum_k \alpha_{k-\vartheta}(z-x)^{-k+\vartheta-1}. \tag{3}$$

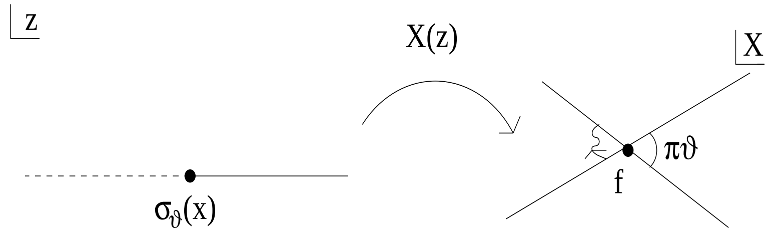

Figure 2. A twisted open string.

Here the worldsheet coordinate, $z = -e^{\tau - i\sigma}$, has domain the upper-half complex plane. This can be extended to the entire complex plane using the 'doubling trick', i.e. we define,

$$\partial X(z) = \begin{cases} \partial X(z) \ \mathrm{Im}(z) \geq 0 \\ \bar{\partial}\bar{X}(\bar{z}) \ \mathrm{Im}(z) < 0 \end{cases}, \tag{4}$$

and similarly for $\partial \bar{X}(z)$. With this extension, the mode expansion in (3) is identical to that of a closed string state in the presence of a $\mathbb{Z}_N$ orbifold twist field [17] (with the replacement $\vartheta = \frac{1}{N}$). Therefore, an open string stretched between two intersecting D-branes is analogous to a twisted closed string state on an orbifold. Hence, we must introduce a twist field $\sigma_\vartheta(x)$[a] for the open string. Such a twist field changes the boundary conditions of X to be those required at the intersection point, $X(x) = f$, of two D-branes. This is achieved by introducing a branch cut in the complex plane, as depicted in 2. We can easily obtain the OPEs,

$$\begin{aligned} \partial X(z)\sigma_\vartheta(x) &\sim (z-x)^{-(1-\vartheta)}\tau_\vartheta(x), \\ \partial \bar{X}(z)\sigma_\vartheta(x) &\sim (z-x)^{-\vartheta}\tau'_\vartheta(x), \end{aligned} \tag{5}$$

where τ'_ϑ and τ_ϑ are excited twists. Also, the local monodromy conditions for transportation around $\sigma_\vartheta(x)$ are,

$$\begin{aligned} \partial X(e^{2\pi i}(z-x)) &= e^{2\pi i\vartheta}\partial X(z-x), \\ \partial \bar{X}(e^{2\pi i}(z-x)) &= e^{-2\pi i\vartheta}\partial \bar{X}(z-x). \end{aligned} \tag{6}$$

These will be employed in the next subsection, where we can now discuss the classical solutions required for a determination of the three point amplitude.

[a] Since we are considering tree level amplitudes, all open string vertices will be conformally mapped onto the real axis.

2.3 Classical solutions and global monodromy

The three point function requires three twist vertices, $\sigma_{\vartheta_i}(x_i)$, corresponding to the three twisted string states at the D-brane intersections. We obtain the asymptotic behaviour of X_{cl} at each of the D-brane intersections from the OPEs (5),

$$\begin{aligned} \partial X(z) &\sim (z-x_i)^{-(1-\vartheta_i)} \text{ as } z \to x_i, \\ \partial \bar{X}(z) &\sim (z-x_i)^{-\vartheta_i} \quad\quad \text{ as } z \to x_i. \end{aligned} \tag{7}$$

Then our classical solutions are determined, up to a normalisation constant to be,

$$\begin{aligned} &\partial X_{cl}(z) = a\omega(z),\ \partial \bar{X}_{cl}(z) = \bar{a}\omega'(z), \\ &\bar{\partial} X_{cl}(\bar{z}) = b\bar{\omega}'(\bar{z}),\ \bar{\partial}\bar{X}_{cl}(\bar{z}) = \bar{b}\bar{\omega}(\bar{z}), \end{aligned} \tag{8}$$

where,

$$\omega(z) = \textstyle\prod_{i=1}^{3}(z-x_i)^{-(1-\vartheta_i)},\ \omega'(z) = \textstyle\prod_{i=1}^{3}(z-x_i)^{-\vartheta_i}. \tag{9}$$

The contribution to S_{cl} coming from the antiholomorphic solution diverges and hence we must set $b = 0$.

We can now determine the remaining normalisation constant from the global monodromy conditions, i.e. the transformation properties of X as it is transported around more than one twist operator, such that the net twist is zero. These conditions are derived from the transformation of X around a single twist vertex, which is,

$$X(e^{2\pi i}z, e^{-2\pi i}\bar{z}) = e^{2\pi i\vartheta}X + (1-e^{2\pi i\vartheta})f, \tag{10}$$

where f is the intersection point of the two D-branes. This can be seen from the local monodromy conditions (6) and the fact that f must be left invariant. The global monodromy of X is then simply a product of such actions.

In this three point case, there exists only a single closed loop, C, with net twist zero. This contour is a Pochammer loop and is depicted in figure 3. Here we have set $x_1 = 0, x_2 = 1$ and $x_3 \to \infty$ using $SL(2,\mathbb{R})$ invariance and the dashed lines denote branch cuts. Evaluating the global monodromy condition,

$$\Delta_C X = \oint_C dz\, \partial X(z) + \oint_C d\bar{z}\, \bar{\partial} X(\bar{z}), \tag{11}$$

with the left hand side being determined by repeated applications of (10) and the right hand side simply by integration, we can determine our normalisation constant a.

Finally, in order to fully calculate S_{cl}, we must employ the methods developed in [18] to express the integral over the complex plane in terms of

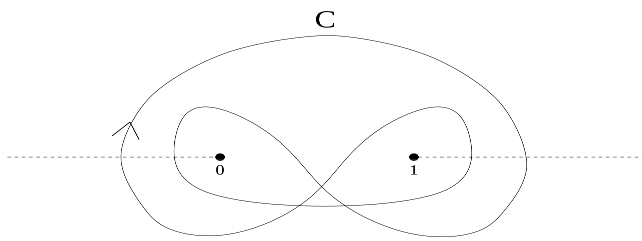

Figure 3. The Pochammer loop.

holomorphic and anti-holomorphic contours. We then determine the classical contribution to the three point amplitude to be,

$$S_{cl} = \frac{1}{2\pi\alpha}\left(\frac{\sin\pi\vartheta_1\sin\pi\vartheta_2}{2\sin\pi\vartheta_3}|f_1 - f_2|^2\right). \tag{12}$$

As commented earlier, this is simply the area of the triangle swept out by the worldsheet. However, our basic methodology can now be extended to the four point amplitude and we have illustrated the fact that the D-brane geometry is encoded in the conformal field theory. Notice that this exponential dependence on worldsheet area leads to a natural mechanism for hierarchy generation as pointed out in [2].

For any order amplitude, we also have contributions coming from worldsheets which wrap the internal space. However, since the classical contribution has the form $e^{-\frac{1}{2\pi\alpha}\text{Area}}$, and the wrapping contributions in general have far larger area then the single unwrapped case, we have determined the leading order contribution to the amplitude.

Finally, to fully determine the three point amplitude we also require the quantum contribution. However, since this is independent of the generation number, it is simply an overall factor in the case of Yukawa interactions. Furthermore, it can be obtained from factorisation of the four point amplitude to which we now proceed.

3 The general four point amplitude

We now outline the calculation of the general four point amplitude, this is required when there are four indpendent sets of D6-branes. The complete calculation can be found in [16].

3.1 Classical contribution to the four point amplitude

Beginning with the classical contribution, we employ an identical methodology to the three point case. The classical solutions we require are,

$$\begin{aligned} &\partial X_{cl}(z) = a\omega(z),\ \partial \bar{X}_{cl}(z) = \bar{a}\omega'(z), \\ &\bar{\partial} X_{cl}(\bar{z}) = b\bar{\omega}'(\bar{z}),\ \bar{\partial}\bar{X}_{cl}(\bar{z}) = \bar{b}\bar{\omega}(\bar{z}), \end{aligned} \tag{13}$$

where,

$$\omega(z) = \textstyle\prod_{i=1}^{4}(z-x_i)^{-(1-\vartheta_i)},\ \omega'(z) = \prod_{i=1}^{4}(z-x_i)^{-\vartheta_i}. \tag{14}$$

However, the antiholomorphic solution now contributes to S_{cl} and we must determine both a and b from the global monodromy conditions. To match this requirement, we now have two independent Pochammer loops to which we apply our global monodromy condition (11). This arises as now there are four twist vertices which can be set to the positions $0, x, 1$ and $x_4 \to \infty$. Hence, we have a Pochammer contour looping around 0 and x and another around x and 1. Therefore, we have two conditions allowing us to determine our two normalisation constants.

As before, we calculate the integrals in S_{cl} using the methods of [18]. Then, combining with the expressions for our normalisation constants, and employing some algebra we obtain,

$$S_{cl}(x) = \frac{\sin(\pi\vartheta_2)}{4\pi\alpha'}\left(\frac{((v_{12}\tau - v_{23})^2 + \gamma\gamma'(v_{12}(\beta+\tau) + v_{23}(1+\alpha\tau))^2)}{(\beta + 2\tau + \alpha\tau^2)}\right), \tag{15}$$

where,

$$\begin{aligned} &\alpha = -\tfrac{\sin(\pi\vartheta_1 + \pi\vartheta_2)}{\sin(\pi\vartheta_1)}, \qquad \beta = -\tfrac{\sin(\pi\vartheta_2+\pi\vartheta_3)}{\sin(\pi\vartheta_3)}, \\ &\gamma\gamma' = \tfrac{\sin(\pi\vartheta_1)\sin(\pi\vartheta_3)}{\sin(\pi\vartheta_2)\sin(\pi\vartheta_4)},\ \tau = \left|\tfrac{F_2}{F_1}\right|, \end{aligned} \tag{16}$$

and

$$F_1 = \textstyle\int_0^x \prod_{j=1}^{3}(y-x_j)^{-(1-\vartheta_j)}dy,\ F_2 = \int_x^1 \prod_{j=1}^{3}(y-x_j)^{-(1-\vartheta_j)}dy. \tag{17}$$

3.2 Quantum contribution to the four point amplitude

We now briefly discuss the quantum part of the four point amplitude. We assume four fermions localised at the intersections of four independent sets of D-branes. The tree-level amplitude is given by a disc diagram with four vertex operators, $V^{(a)}$, in the -1/2 picture. Using $SL(2,\mathbb{R})$ invariance, we write the ordered amplitude as,

$$\begin{aligned} &(2\pi)^4\delta^4(\textstyle\sum_a k_a)A(1,2,3,4) = \\ &\tfrac{-i}{g_s l_s^4}\textstyle\int_0^1 dx\langle V^{(1)}(0,k_1)V^{(2)}(x,k_2)V^{(3)}(1,k_3)V^{(4)}(\infty,k_4)\rangle. \end{aligned} \tag{18}$$

The required vertex operators for the fermions are,

$$V_i^{(a)}(x_a, k_a) = const\ \lambda^a u_\alpha^{(i)} S_i^\alpha \sigma_{\vartheta_i} e^{-\phi/2} e^{ik_a \cdot X}(x_a), \tag{19}$$

where u_α is the space time spinor polarization, S^α is the spin-twist operator and $e^{-\phi/2}$ is the contribution from the superconformal ghosts. The spin-twist operator arises from bosonization of the fermionic worldsheet fields and is of the form,

$$S_i^\alpha = \prod_{l=1}^{5} : \exp(iq_i^l H_l) : \tag{20}$$

where for D6-branes intersecting at angles we have,

$$q_i^l = \left(\pm\frac{1}{2}, \pm\frac{1}{2}, \vartheta_i^1 - \frac{1}{2}, \vartheta_i^2 - \frac{1}{2}, \vartheta_i^3 - \frac{1}{2}\right). \tag{21}$$

The relative sign of the first two entries determines the helicity of the fermion, and $\vartheta_i^{m=1,2,3}$ are the angles of the i'th intersection of the D-branes in the m'th complex plane.

The correlation function can be computed using OPEs and wick contraction. However, we run into the problem of not being able to determine the OPEs of two twist fields. We circumvent this using a method developed in [17]. In outline, we determine the correlation function of four twist vertices by first calculating,

$$\frac{\langle T(z) \prod_{i=1}^4 \sigma_{\vartheta_i} \rangle}{\langle \prod_{i=1}^4 \sigma_{\vartheta_i} \rangle} = \lim_{w \to z} [g(z, w) - \frac{1}{(z-w)^2}]. \tag{22}$$

Here $g(z, w)$ is the usual Green function for the closed string, whose functional form may be determined from various asymptotics. We also know the OPE of the stress energy tensor, $T(z)$, with a twist field. Combining these expressions gives rise to a set of differential equations for $\langle \prod_{i=1}^4 \sigma_{\vartheta_i} \rangle$, containing a term with an unknown constant. This may be determined from global monodromy conditions, resulting in,

$$\langle \prod_i \sigma_{\vartheta_i} \rangle = |J|^{-\frac{1}{2}} x_\infty^{-\vartheta_4(1-\vartheta_4)} x^{\frac{1}{2}(\vartheta_1+\vartheta_2-1)-\vartheta_1\vartheta_2} (1-x)^{\frac{1}{2}(\vartheta_2+\vartheta_3-1)-\vartheta_2\vartheta_3}, \tag{23}$$

where $|J| = |F_1||F_2'| + |F_2||F_1'|$ and F_i' is obtained from F_i by the substitution $\vartheta_i \to 1 - \vartheta_i$.

Piecing together the correlation functions of all the relevant fields and including contributions from each torus subfactor, we obtain the full four

point amplitude,

$$\begin{aligned}&A(1,2,3,4) = -g_s\alpha' Tr[\lambda^1\lambda^2\lambda^3\lambda^4 + \lambda^4\lambda^3\lambda^2\lambda^1]\\ &\int_0^1 dx\, x^{-1-\alpha' s}(1-x)^{-1-\alpha' t}\frac{1}{\prod_m^3 |F_1^m \bar{F'}_2^m - \bar{F'}_1^m F_2^m|^{1/2}}\\ &\times\left[\bar{u}^{(2)}\gamma_\mu u^{(1)}\bar{u}^{(4)}\gamma^\mu u^{(3)}\right]\sum e^{-S_{cl}(x)},\end{aligned} \tag{24}$$

where,

$$F_i^m = \int_{x_i}^{x_{i+1}} (y)^{-(1-\vartheta_1^m)}(y-x)^{-(1-\vartheta_2^m)}(y-1)^{-(1-\vartheta_3^m)}dy. \tag{25}$$

and $F_i'^m$ is again obtained from F_i^m by the substitution $\vartheta_i \to 1-\vartheta_i$.

4 Summary and conclusions

We have outlined the techniques required to determine interactions between states localised at D-brane intersections. This was illustrated with an application to Yukawa interactions. We have also performed the first calculation of the general four point amplitude. This result is required in the case of four independent stacks of D-branes with the relevant interaction being of the form $q_L q_R \to e_L e_R$. Our results can also be applied to various phenomenological issues such as flavour changing neutral currents [19] and proton decay [20].

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